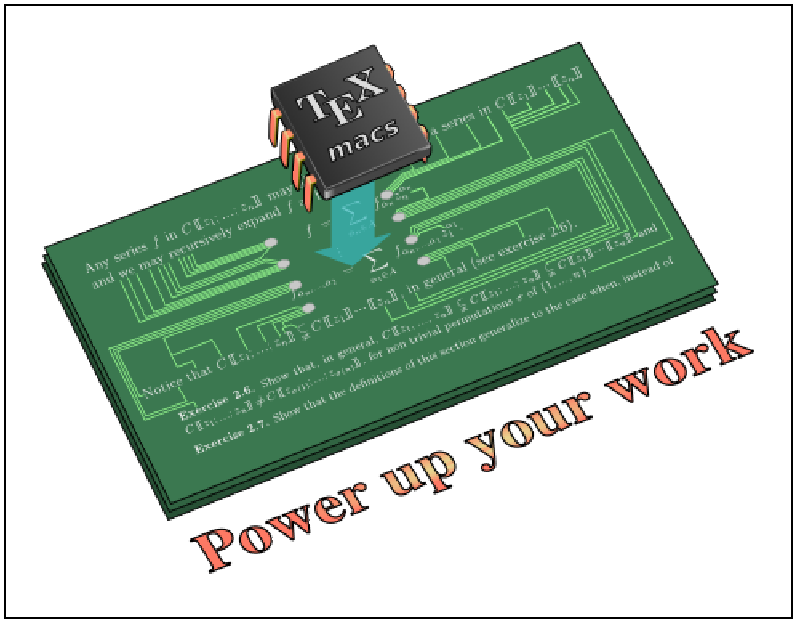
Towards semantic mathematical editing



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http://www.TEXMACS.org

Three levels of mathematical documents

Informal mathematics.

Software: text editors, mathematical user interfaces

Formats: LaTeX, presentation MathML

Example: `$a(b+c)$` LaTeX

Example: Presentation MathML

```
<mrow>
  <mi>a</mi> <mo>&InvisibleTimes;</mo>
  <mo>(</mo> <mi>b</mi> <mo>+</mo> <mi>c</mi> <mo>)</mo>
</mrow>
```

Example: a*⟨around|(|b+c|)⟩ TeXmacs

Syntactically correct documents.

Software: computer algebra systems, scientific computation systems

Formats: content MathML, software specific languages

Example: Content MathML

```
<apply>
  <times/>
  <ci>a<ci>
  <apply>
    <plus/> <ci>b</ci> <ci>c</ci>
  </apply>
</apply>
```

Example: `(* (+ a b))` Scheme

Semantically correct documents.

Software: automatic theorem provers/checkers

Formats: OpenMath, OMDoc, software specific languages

Example: $a\,(b+c)$, where $a, b, c \in \mathbb{Z}$ and $+, \cdot : \mathbb{Z}^2 \to \mathbb{Z}$

The challenge

General purpose user interfaces.

Presentation oriented, no syntactic or semantic correctness.

Improved general purpose interfaces.

Presentation oriented interface *while* enforcing syntactic correctness

Improved general purpose interfaces.

Presentation oriented interface *while* enforcing syntactic correctness

Interfaces for special systems.

May enforce syntactic or semantic correctness, application specific

Possible approaches

User friendly extreme.

Rely on intelligent software to make sense out of presentation markup.

Programmer friendly extreme.

Let the user provide the explicit content markup.

Compromise.

- Make as much sense out of presentation markup as possible:
  → Automatic upgrading
  → Automatic syntax correction
  → Packrat parsing
- Provide markup for correcting the default interpretation, when needed.

Central technique.

→ Presentation oriented interface
→ Background converter presentation markup ⇝ content markup

Common ambiguities

Invisible operators.

- Invisible separators: $A = (a_{ij})$
- Invisible addition: $17\,{}^{3}\!/_{8}$
- Invisible "wildcards": $+1$ (also denoted by $\cdot + 1$)
- Invisible ellipses: $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$
- Invisible zeros: $\begin{pmatrix} a_{11} & & \\ & \ddots & \\ & & a_{nn} \end{pmatrix}$
- Invisible brackets, for forced matching if we want to omit a bracket

Vertical bars.

- Brackets: absolute values $|x|$ or "ket" notation $\langle A|$
- "Such that" separators: $\{a \in X \,|\, a \geqslant 0\}$
- "Divides" predicate: $11 \,|\, 1001$
- Separators: $\langle a_1 | \cdots | a_n \rangle$.
- Restricting domains or images: $\varphi|_D$, $\varphi|^I$.

Ponctuation.

- Commas: $f(x, y)$, $3{,}14159\cdots$, $1{,}000{,}000$
- Periods: 3.14159, $\lambda x.x^2$, data access $a.b$, period.
- Semicolons: $\{x \in X : x \geqslant 0\}$, $P = (x : y : z) \in \mathbb{P}^2$, $a : \mathsf{Int}$.
- Moreover, in the formula

$$a^2 + b^2 = c^2, \tag{1}$$

  ponctuation is used in the traditional, non-mathematical way.

Miscellaneous homoglyphs.

- Backslash $\setminus$: separator or "subtraction" of sets $X \setminus Y$
- Dot $\cdot$: multiplication $\boldsymbol{a} \cdot \boldsymbol{b}$ or wildcard $|\cdot|_p$
- Wedge $\wedge$: logical and $P \wedge Q$ or exterior product $\mathrm{d}x \wedge \mathrm{d}y$.

Good news.

THAT IS ABOUT IT! ...but inconsistent support in Unicode

Syntax correction

Invisible tag correction.

Breaks: `\begin{math}a+\end{math}\begin{math}b\end{math}`.

Redundancies I: `\begin{math}a+\begin{math}b\end{math}\end{math}`.

Redundancies II: `\begin{math}\text{hi}\end{math}`.

Bracket matching. Match in decreasing order of likeliness

Example: $f(|x|) = g(|x| + |y|)$

Match $(|x|)$ and $(|x| + |y|)$ at an early stage.

Match $|x|$, $|x|$ and $|y|$ at a later stage, with higher security

Bracket motion. `Let $y=f(x$).` ⇝ `Let $y=f(x)$.`

Superfluous invisible removal. `$\frac{a\*}{b}$` ⇝ `$\frac{a}{b}$`

Missing invisible insertion. `$2x$` ⇝ `$2\*x$`, `$Lf$` ⇝ `$L\_f$`.

Homoglyph substitution. $a :=b$ ⇝ $a := b$

Miscellaneous corrections. `$a^{^x}$` ⇝ `$a^x$`

Experimental results

BPR: Algorithms in Real Algebraic Geometry (Basu, Pollack, Roy)
COL: Collection of class notes (Evan Chou)
LN: Transseries and Real Differential Algebra (vdH)

Document	BPR	BPR_1	BPR_2	COL	COL_3	LN
Total # of formulas	30394	883	2693	13048	2092	12626
Initial # of errors	2821	63	221	4158	607	629
# after correction	705	35	53	543	37	98
Number of pages	585	16	48	357	56	233

Table. Performance of the TEX_{MACS} syntax corrector on various documents.

Demo 1: BPR_2

Demo 2: $Arxiv_1$

BPR: Algorithms in Real Algebraic Geometry (Basu, Pollack, Roy)
COL: Collection of class notes (Evan Chou)
LN: Transseries and Real Differential Algebra (vdH)

Document	BPR	BPR_1	BPR_2	COL	COL_3	LN
Invisible tag correction	92	0	3	10	5	56
Bracket matching	676	14	67	1236	94	282
Bracket motion	16	0	4	6	0	4
Superfluous invisible removal	382	1	25	1046	305	149
Missing invisible insertion	873	11	56	1271	164	33
Homoglyph substitution	44	2	7	45	2	6
Miscellaneous corrections	16	0	6	1	0	1

Table. Numbers of corrections due to individual algorithms.

BPR: Algorithms in Real Algebraic Geometry (Basu, Pollack, Roy)
COL: Collection of class notes (Evan Chou)
HAB: Habilitation (vdH)

Document	BPR_1	COL_3	HAB_1	HAB_5
Invisible operator confusion	some	many		some
Informal list notation	several		several	some
Non marked text inside formulas	several	many		
Non marked formulas inside text	some			
Misinterpreted meaningful whitespace			several	
Miscellaneous misinterpretations		some		

Table. Manual determination of common sources of misinterpretation.

Parsing informal mathematics

- **What kind of Parser?**
  Packrat parsers are fast ($O(n\,s)$ parsing, n: #input, s: #grammar)
  Packrat parsers are flexible (no preprocessing required)
  Packrat parsers are general (better set of recognized languages)
- **What kind of grammar?**
  Non-structured string grammar $\Rightarrow$ flattening of structured texts
  Universal grammar for mathematics
  Might support other grammars for (e.g.) automatic theorem provers
- **How to allow for customization?**
  Exploit TEXMACS built-in macro system
  Allow for "behaves as" annotations

Snippets of the universal grammar

```
(define Plus-symbol
  (:type infix)
  (:penalty 30)
  (:spacing default default)
  "+" "<amalg>" "<oplus>" "<boxplus>"
  "<dotplus>" "<dotamalg>" "<dotoplus>")
```

```
(define Plus-infix
  (:operator associative)
  (Plus-infix Post)
  (Pre Plus-infix)
  Plus-symbol)
```

```
(define Pre
  (:selectable inside)
  (:<lsub Script :>)
  (:<lsup Script :>)
  (:<lprime (* Prime-symbol) :>))

(define Post
  (:selectable inside)
  (:<rsub Script :>)
  (:<rsup Script :>)
  (:<rprime (* Prime-symbol) :>))
```

```
(define Sum
  (Sum Plus-infix Product)
  (Sum Minus-infix Product)
  Sum-prefix)
```

How much structure in the document?

Brackets.

Previously: no structure in document $f(x+y)+a\,(b+c)$

Currently: around tag $f(x+y)+a\,(b+c)$

Subscripts and superscripts.

Base not in document

$\hookrightarrow$ either requires parsing or hard to edit

Scripts physically glued to content on direct left $a+\lim_{i\to\infty} x_i$

Big operators.

Scope not in document

Prefix operators, arguments variable priorities $\sum_{i=1}^{n} a_i + \sum_{i=1}^{p} b_i\,c_i$

Grammar rules for big operators

```
(define Big-sum-symbol
  "int" "oint" "intlim" "ointlim" "sum" "oplus" "triangledown")

(define Big-sum
  (:operator)
  (Big-sum Post)
  (:<big Big-sum-symbol :>))

(define Prefixed
  (Big-separator Expression)
  (Big-or Conjunction)
  (Big-and Negation)
  (Big-union Intersection)
  (Big-intersection Sum)
  (Big-sum Sum-prefix)
  (Big-product Power)
  (Prefix-prefix Prefixed)
  (Pre Prefixed)
  (Postfixed Space-infix Prefixed)
  Postfixed)
```

Customization

"Behaves as" annotation.

$$a_1 + \cdots\cdots + a_n$$

Macro expansion.

```
⟨assign|pt|⟨macro|⟨math-times|⟨superpose|+|<times>⟩⟩⟩⟩
```

$$a * b + x * y$$

Customizing the interface.

```
Scheme] (kbd-map (:mode in-math?) ("+ *" (make 'pt)))
Scheme]
```

Non parsable formulas

Meaningful whitespace or text.

$$\exists x \quad P(x)$$

$$\left\{ b < \mathbb{N}_0 \middle| b < d \text{ and } \frac{\det A}{d}\,\mathbb{Z} + b\,\mathbb{Z} + d\,\mathbb{Z} = \mathfrak{d}_1(A) \right\}$$

Using mathematics as a replacement for text.

$$a) \Rightarrow b)$$

Manual hyphenation on tables.

$$\begin{aligned} \mathrm{posgcd}(\mathcal{P} \cup \{P\}) \;\; = \;\; & \{(\mathrm{Pol}(p(L)), \mathcal{C} \wedge \mathcal{C}_L) |\, (Q, \mathcal{C}) \in \mathrm{posgcd}(\mathcal{P}) \\ & \text{and } L \text{ leaf of } \mathrm{TRems}(P, Q)\}. \end{aligned}$$

Unsuccessful automatic correction.

$$\mathrm{T}^{j}_{*}\nu$$

$$\$a+\$ \;\; \$b\$$$

$$\mathcal{B} = \left\{ a + i\,b \middle|\, |b| \leqslant -\sqrt{3}\,a \right\}$$

Non standard operator precedence.

Given a formula $\Theta(Y) = (\exists X)\ \mathcal{B}(X, Y)$, where $\mathcal{B}$ is...

Abusive visual twiddling.

$$\prod_{\substack{i<j,\,k<\ell \\ (i,j)<(k,\ell)}} (\alpha_{i,j,k,\ell} + Z\,\beta_{i,j,k,\ell})^2$$

Visual twiddling with no obvious alternative.

1. The signs of the polynomials in the Sturm sequence are $+-+-+$
2. No other code word is of the form $0.z_1 \ldots z_{l(x)} * \ldots$
3. $g = \mathrm{e}^{\sqrt{x} + \mathrm{e}^{\sqrt{\log x} + \mathrm{e}^{\sqrt{\log\log x} + \cdot^{\cdot^{\cdot}} + \log\log\log x} + \log\log x} + \log x}$

Incorrectly parsed formulas

Incorrect invisible operators.

1. $b(12\,c + a^2)$
2. $\sim \mathsf{R}(n)\,s$

Informal list notation.

1. $\mathrm{SRemS}(P, Q) = \mathrm{SRemS}_0(P, Q), \ldots, \mathrm{SRemS}_k(P, Q)$
2. The subsets of affine spaces $\mathbb{R}^n$ for $n = 0, 1, 2, \ldots$ that are...
3. $H(Y_i|Y^{i-1}, X^n)$

Non marked text inside formulas.

$p(\hat{x}\,|x) = 1 \text{ if } \hat{x} = 0$

Misinterpretation of meaningful whitespace.

$P(f) = 0 \qquad (f \prec \mathfrak{V}).$

Miscellaneous misinterpretations.

$$\max_{p,q,r} -1 - p\log p - q\log q - r\log r - (1-p-q-r)\log(1-p-q-r)$$

Semantic selections

Selectable subexpressions.

Associative operators

$$a+b-c+d$$

Selections as unary operators when pasting

$$+a+b \quad \leftrightsquigarrow \quad x \longmapsto x+a+b$$

Second example

$$\frac{1}{1+} \quad \leftrightsquigarrow \quad x \longmapsto \frac{1}{1+x}$$

Problematic transformations.

$$a_1, \ldots, a_n \in A \quad \rightsquigarrow \quad (a_1, \ldots, a_n) \in A$$

Semantic typesetting

Spacing.

1. Binary subtraction $x - y$
2. Unary negation $-x$
3. Abstract operators $\square \in \{+, -, \cdot\}$

Hyphenation transforms.

$$\frac{a+b+c}{x+y+z} \rightsquigarrow (a+b+c)/(x+y+z)$$

$$\frac{1}{a+b+c+x+y+z} \rightsquigarrow 1/(a+b+c+x+y+z)$$