PhD proposal

Title: Numerical approach of generalized flatness

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Contacts

Joris van der Hoeven <vdhoeven@lix.polytechnique.fr>
François Ollivier <francois.ollivier@lix.polytechnique.fr>

Address

Laboratoire d'informatique de l'École polytechnique, LIX, UMR 7161 CNRS
Campus de l'École polytechnique, Bâtiment Alan Turing, CS35003
1 rue Honoré d'Estienne d'Orves
91120 Palaiseau, France

Director of the laboratory: Mr Gilles Schaeffer (schaeffe@lix.polytechnique.fr)

Research team: MAX, Algebraic modeling and symbolic computation

M2 students majoring in computer science or mathematics may apply. Knowledge in at least one of the following topics is required: computer algebra, differential algebra, control theory, numerical analysis. Basic programming skills are also appreciated.

Context

The MAX team is searching for PhD candidates on the themes of the ERC ODELIX project: *solving differential equations fast, precisely, and reliably*. The present proposal concerns applications of these themes to control theory.

Description

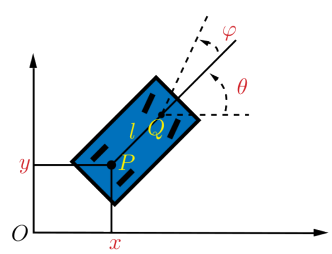
Consider a dynamical system model, described through a parametric system of ordinary differential equations (ODEs)

$$x' = f(x, u),$$

where x is a vector of state functions and u is a vector of control functions. This system is said to be *flat* [1] if there exists a local parametrization $x_i = X_i(z_1, \ldots, z_m)$, where X_i is a function of the functions z_j and of a finite number of their derivatives, and the z_j are functions of the time that may be chosen arbitrarily. Furthermore, the z_j must be expressed as differential functions of the state and the control: $z_j = Z_j(x, u)$. These functions z_j are called *flat outputs*.

Such a property greatly facilitates *motion planning*, that is to design a control to go from a given starting point to a given goal.

A car is a classical example of a flat system, for which the coordinates of any point of the rear axis is a flat output.



Generic systems are not flat, but flatness is ubiquitous in engineering [3]. This may require some simplifications of the model. For examples, a classical aircraft model is flat if one neglects the thrusts created by the actuators (ailerons, elevators, rudder) [4]. Flatness makes also easier to design a feed-back that can compensate perturbations but also model errors [2].

In a recent paper, it has been proposed to improve the flat control of an aircraft by using the values of the thrusts due to the actuators, provided by the classical flat parametrization, to design a new more accurate parametrization [5]. This process can be iterated to produce a very precise motion planning for the aircraft model without simplification.

The order of derivatives of the flat outputs required for such a parametrization increases with the number of iterations, so that this generalized flat parametrization depends potentially of an infinite number of derivatives. One may conjecture that all systems would be flat when allowing functions involving an infinite number of derivatives.

The goal is to investigate this notion of *generalized flatness*. According to the taste and skills of candidates, this may be done in many ways, focusing on computer experiments, or theoretical investigations. Obviously some investment on computer simulations will be required first to get an intuition relying on practical knowledge. On may consider for example a car with two trailers, that is flat if the trailers are attached just above the rear axis, but not in the general case [6]. A generalized flat parametrization may be designed using iterations or homotopy methods, that is slowly moving the points where trailers are attached.

One may also notice that generalized flatness allows to parametrize flat systems, using a function that is not a genuine flat output. For example, y is a flat output for the system $x = y'$ but not for the system $x - \epsilon x' = y'$. Anyway, one may use for small ϵ the parametrization $x = \sum_{i \in \mathbb{N}} \epsilon^i y^{(i+1)}$.

One may take advantage of existing implementations in Maple for the aircraft and use the more efficient tools provided by Mathemagix. The field of possible applications is large and include all non flat systems and all flat ones for which one would like to use alternative sets of functions that are not actual flat outputs.

The range of theoretical investigations is also very large, starting with proofs of convergence. One may also consider the possible unicity of this generalized flat parametrization.

Expected results

Firstly, it will be necessary to deal with classical examples, such as cars with trailers, through the creation of dedicated implementations. This work should result in one or more publications.

Work more oriented towards computer science should include the creation of efficient generic code, covering a large class of examples, accompanied by a complexity analysis, benchmarks, and a study of the precision of the results obtained. This will also give rise to one or more publications.

We expect a minimal contribution to the theoretical setting, at least in the linear case. Work more oriented towards the mathematical aspects of control should focus on providing precise definitions and reporting experimental behavior through proven statements, related to the convergence of calculations and their precision.

Of course, the above list is purely indicative and may be adapted as a function of the profile of the candidate. The software will be distributed under a free software license and publications will take place in prominent journals or conference proceedings of the area, such as JSC, AAECC, ISSAC, etc.

Bibliography

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